Math 481: Vector and Tensor Analysis (3 credits)

Course Description

Introductory course in modern differential geometry focusing on examples, broadly aimed at students in mathematics, the sciences, and engineering. Emphasis on rigorously presented concepts, tools and ideas rather than on proofs. The topics covered include differentiable manifolds, tangent spaces and orientability; vector and tensor fields; differential forms; integration on manifolds and Generalized Stokes Theorem; Riemannian metrics, Riemannian connections and geodesics. Applications to configuration and phase spaces, Maxwell equations and relativity theory will be discussed.

Prerequisite: MATH 241 and one of MATH 415 or MATH 416 or equivalent.

Course Objectives

Students should leave with a working knowledge and examples of smooth manifolds. After completing the course, they should see connections between vector fields, differential equations and tangent vectors and gain experience with differential forms and exterior algebra. In addition, students should be able to determine how methods from linear algebra and calculus can be used to study geometric objects, surfaces, manifolds and geodesics.

Course Content

1. **Manifolds**
   - Abstract differentiable manifolds
   - Tangent Spaces
   - Tangent Bundles
   - Orientability

2. **Calculus on Manifolds**
   - **Vector Fields**
   - **Flows**
   - Tensor Fields

3. **Differential Forms and Exterior Calculus**

4. **Singular Cubes and Singular Chains**
   - **Integration Theory on manifolds**
   - **Generalized Stokes' theorem**

5. **Riemannian Geometry**
   - **Riemannian metrics**
   - **Riemannian connections**
   - **Geodesics**
   - **Curvature**

Format

- This is an online course featuring video lectures from the UIUC Spring 2017 course taught by Professor Ely Kerman.
- Text: Theodore Frankel. (1997). *The Geometry of Physics, An Introduction* (3rd Edition). Cambridge.
- Students must be able to print out assignments, write out solutions, then scan their written work and upload it to Moodle to meet set deadlines.
- This course requires multiple paper-based exams that must be taken with an approved proctor. Exams may be taken on campus with NetMath proctoring; for off-campus options see https://netmath.illinois.edu/offcampus. Off-campus proctors must be able to scan completed exams and email them to NetMath for grading, as well as mailing the paper exam back for archival purposes.