Math 448: Complex Variables (3 credits)

Course Description

This course is for students who desire a rigorous introduction to the theory of functions of a complex variable. Topics include Cauchy's theorem, the residue theorem, the maximum modulus theorem, Laurent series, the fundamental theorem of algebra, and the argument principle.

Prerequisite: MATH 447

Course Objectives

The course covers the first three chapters of the text *Complex Variables* by Stephen D. Fisher. Students should leave with a fundamental knowledge of the basic theory and applications of complex variables through reading, understanding and constructing proofs. It is important to recognize that reading ahead in the textbook before viewing class videos will make the lectures more comprehensible and writing your homework solutions in your own words improves your understanding.

Course Content

1. The Complex Plane

Complex numbers, complex plane
Triangle inequality
De Moivre's Theorem
Algebraic construction of complex numbers
Geometry of the complex plane, topology of the complex plane
Functions and limits, infinite series
Exponential, logarithm, and trigonometric functions
Line integrals and Green's theorem

2. Basic Properties of Analytic Functions

Complex differentiability
Analytic and harmonic functions; Cauchy-Riemann equations
Power series
Cauchy's theorem and Cauchy's formula
Consequences of Cauchy's formula: The coefficient of power series, Liouville's Theorem
The order of zero
Isolated singularities: removable singularities, poles, and essential singularities
Laurent series
Computation of residues
Residue theorem and evaluation of definite integrals and infinite sums

3. Analytic Functions as Mappings

Zeros of an analytic function
The Argument Principle
Rouché's Theorem
The Fundamental Theorem of Algebra
Maximum Modulus Principle
Schwarz's Lemma
Linear fractional transformations

Format

- This is an online course featuring video lectures from the UIUC Spring 2017 course taught by Professor Bruce Reznick.
- Text: Stephen D. Fisher. (1999). *Complex Variables* (2nd Edition). Dover.
- Students must be able to print out assignments, write out solutions, then scan their written work and upload it to Moodle to meet set deadlines.
- This course requires multiple paper-based exams that must be taken with an approved proctor. Exams may be taken on campus with NetMath proctoring; for off-campus options see https://netmath.illinois.edu/offcampus. Off-campus proctors must be able to scan completed exams and email them to NetMath for grading, as well as mailing the paper exam back for archival purposes.