Math 448 XGR: Complex Variables (3 credits)
Note: This section is restricted to students enrolled in UIUC Online Engineering Graduate Degree Programs.

Course Description

This course is for students who desire a rigorous introduction to the theory of functions of a complex variable. Topics include Cauchy's theorem, the residue theorem, the maximum modulus theorem, Laurent series, the fundamental theorem of algebra, and the argument principle.

Prerequisite: MATH 447

Course Objectives

The course covers the first three chapters of the text *Complex Variables* by Stephen D. Fisher. Students should leave with a fundamental knowledge of the basic theory and applications of complex variables through reading, understanding and constructing proofs. It is important to recognize that reading ahead in the textbook before viewing class videos will make the lectures more comprehensible and writing your homework solutions in your own words improves your understanding.

Course Content

1. The Complex Plane
- Complex numbers, complex plane
- Triangle inequality
- De Moivre's Theorem
- Algebraic construction of complex numbers
- Geometry of the complex plane, topology of the complex plane
- Functions and limits, infinite series
- Exponential, logarithm, and trigonometric functions
- Line integrals and Green's theorem

2. Basic Properties of Analytic Functions
- Complex differentiability
- Analytic and harmonic functions; Cauchy-Riemann equations
- Power series
- Cauchy's theorem and Cauchy's formula
- Consequences of Cauchy's formula: The coefficient of power series, Liouville's Theorem
- The order of zero
- Isolated singularities: removable singularities, poles, and essential singularities
- Laurent series
- Computation of residues
- Residue theorem and evaluation of definite integrals and infinite sums

3. Analytic Functions as Mappings

Zeros of an analytic function
The Argument Principle
Rouché's Theorem
The Fundamental Theorem of Algebra
Maximum Modulus Principle
Schwarz's Lemma
Linear fractional transformations

Format

- This is an online course featuring video lectures from the UIUC Spring 2017 course taught by Professor Bruce Reznick.
- Text: Stephen D. Fisher. (1999). *Complex Variables* (2nd Edition). Dover.
- Students must be able to print out assignments, write out solutions, then scan their written work and upload it to Moodle to meet set deadlines.
- This course requires multiple paper-based exams that must be taken with an approved proctor. Please refer to the Engineering Online Proctor Information Page for proctor eligibility criteria and instructions on how to submit your proctor information.
- Detailed information about how to access exams will be available within the course site.