

DEPARTMENT OF MATHEMATICS



**Math 447: Real Variables (3 credits)**

**Course Description:**

Careful development of elementary real analysis for those who intend to take graduate courses in Mathematics. Topics include completeness property of the real number system; basic topological properties of  $n$ -dimensional space; convergence of numerical sequences and series of functions; properties of continuous functions; and basic theorems concerning differentiation and Riemann integration. Credit is not given for both MATH 447 and either MATH 424 or MATH 444.

Prerequisite: MATH 241 or equivalent; junior standing; MATH 347 or MATH 348 Course

**Objectives:**

Introduction to real analysis is a gateway. The idea is to find balance between rigorous proofs and real understanding. This principle is the core of mathematics at all levels. Be prepared to learn to write proofs. Be prepared to accept a little abstract but clarifying approach to well known, and not so well known topics related to calculus.

**Course Content:**

1. Real Numbers

- Natural numbers
- Abelian groups
- Grothendieck's construction
- Integers
- Fields
- Rational numbers
- Ordered fields
- Completeness
- Peano's axiom
- Uncountability of real numbers

2. Sequences

- Limits
- Monotone sequences
- Subsequences
- Bolzano-Weierstrass
- Limsup and liminf
- Application to continuous functions

### 3. Metric Spaces

Metric spaces  
Cauchy sequences  
Completeness  
Sequential compactness and total boundedness  
Open, closed and compact sets  
Application to Heine-Borel and continuity of inverses  
Connectes sets  
Intermediate value theorem

### 4. Spaces of Continuous Functions

Uniform continuity  
 $C(K)$  is a complete metric space  
Dini's theorem, application  
Interchanging differentiation and limit

### 5. Differentiation

Rolle's lemma and the mean value theorem  
Differentiation of power series

### 6. Integration

Definition  
Interchanging limits  
Fundamental theorem and application to power series

#### Format:

- This is an online course featuring video lectures from the UIUC Spring 2018 course taught by Professor Marius Junge.
- Text: Kenneth Ross. (2013). *Elementary Analysis: The Theory of Calculus* (2nd Edition). Springer.
- Students must be able to view assignments online, write out solutions, then scan or take a photo of their written work and upload it to Moodle.
- This course requires multiple proctored exams.