



Math 447: Real Variables (3 credits)

Course Description:

Careful development of elementary real analysis for those who intend to take graduate courses in Mathematics. Topics include completeness property of the real number system; basic topological properties of n -dimensional space; convergence of numerical sequences and series of functions; properties of continuous functions; and basic theorems concerning differentiation and Riemann integration. Credit is not given for both MATH 447 and either MATH 424 or MATH 444.

Prerequisite: MATH 241 or equivalent; junior standing; MATH 347 or MATH 348 Course

Objectives:

Introduction to real analysis is a gateway. The idea is to find balance between rigorous proofs and real understanding. This principle is the core of mathematics at all levels. Be prepared to learn to write proofs. Be prepared to accept a little abstract but clarifying approach to well known, and not so well known topics related to calculus.

Course Content:

1. Real Numbers

*Natural numbers
Abelian groups
Grothendieck's construction
Integers
Fields
Rational numbers
Ordered fields
Completeness
Peano's axiom
Uncountability of real numbers*

2. Sequences

*Limits
Monotone sequences
Subsequences
Bolzano-Weierstrass
Limsup and liminf
Application to continuous functions*



3. Metric Spaces

Metric spaces
Cauchy sequences
Completeness
Sequential compactness and total boundedness
Open, closed and compact sets
Application to Heine-Borel and continuity of inverses
Connectes sets
Intermediate value theorem

4. Spaces of Continuous Functions

Uniform continuity
 $C(K)$ is a complete metric space
Dini's theorem, application
Interchanging differentiation and limit

5. Differentiation

Rolle's lemma and the mean value theorem
Differentiation of power series

6. Integration

Definition
Interchanging limits
Fundamental theorem and application to power series

Format:

- This is an online course featuring video lectures from the UIUC Spring 2018 course taught by Professor Marius Junge.
- Text: Kenneth Ross. (2013). *Elementary Analysis: The Theory of Calculus* (2nd Edition). Springer.
- Students must be able to view assignments online, write out solutions, then scan or take a photo of their written work and upload it to Moodle.
- This course requires multiple exams that may be taken online.