



Math 423: Differential Geometry: (3 Credits)

Course Description:

This course covers applications of the calculus to the study of the shape and curvature of curves and surfaces; introduction to vector fields, differential forms on Euclidean spaces, and the method of moving frames for low-dimensional differential geometry.

Prerequisite: MATH 241 or equivalent

Course Objectives:

The objectives are the study of curves using multivariable calculus and the study of surfaces using exterior calculus. The course is essentially the study of the concept of curvature. Students are also expected to improve ability in writing mathematical arguments.

Course Content:

1. The Geometry of Curves
 - Basic notions of the theory of curves: regular curves, tangent lines, arc length, parameterization by arc length
 - Plane curves: curvature, Frenet frame
 - Space curves: curvature and torsion, Frenet frames, canonical form of a curve up to rigid motion
 - Hopf's Umlaufsatz
 - The Four-vertex theorem
 - Total curvature

2. Exterior Calculus
 - 1-form and differential of a function
 - Tensors in a vector space: alternating tensors
 - Wedge products
 - Differential k-forms
 - Exterior derivatives
 - Covariant derivatives
 - Regular maps

3. Classical Surface Theory
 - Regular surfaces
 - The tangent plane
 - The first fundamental form
 - Normal fields and orientation of surfaces



The Gauss map
The second fundamental form
Curvature: principal curvature, Gaussian and mean curvatures
Surface area and integration on surfaces
Examples: ruled surfaces, surfaces of revolution, minimal surfaces

4. Intrinsic Surface Theory

Isometries
The fundamental theorem of surfaces (Bonnet's theorem)
Covariant derivatives
Gauss's Theorema Egregium
Parallel transport
Geodesics and the exponential map
The Euler-Lagrange equation
The Gauss-Bonnet theorem and applications (Poincare index theorem)

Format:

- This is an online course featuring video lectures from the UIUC Fall 2017 course taught by Professor Anil Hirani.
- Text: Barrett O'Neill. (2006). *Elementary Differential Geometry* (2nd Edition). Academic Press (Elsevier).
- Students must be able to view assignments online, write out solutions, then scan or take a photo of their written work and upload it to Moodle.
- This course requires multiple exams that may be taken online.