NetMath® DEPARTMENT OF MATHEMATICS

MATH 417: Introduction to Abstract Algebra (3 credit hours)

Course description

Math 417 is an introduction to abstract algebra. The main objects of study are groups, which are abstract mathematical objects that reflect the most basic features of many other mathematical constructions. We will also study rings and fields and other abstract mathematical objects, which can be thought of as groups with additional structure.

Prerequisite: Either MATH 416 or one of MATH 410, MATH 415 together with one of MATH 347, MATH 348, CS 374; or consent of instructor.

Course Objectives

The goal of the course is to introduce students to abstract mathematical thinking through the study of these simple, beautiful mathematical constructions, and to explore the relationship to other areas of mathematics.

Course Content

1. Preliminaries Sets and functions. Equivalence relations. Permutations and cyclic decomposition. The Euclidean algorithm for integers. Greatest common divisor. Congruence arithmetic (Modular arithmetic).

2. Abstract Algebra Fundamental theorem of algebra. Fields: the definition, subfields and extension fields. The Euclidean algorithm for polynomials. Greatest common divisor for polynomials. Vector spaces. Euclidean geometry. Group: the definition and examples. Rings: the definition and examples.

3. Group Theory Subgroups and cyclic groups. Centralizers, Normal subgroups. Normalizers. Cayley’s theorem. Rubik’s Cube. Even and odd permutations. Dihedral groups. Cosets and Lagrange’s theorem. Quotient groups and the isomorphism theorems. Direct products and semidirect products.

4. Group Actions Examples: representations, left regular actions, conjugation actions. Coset actions. Class equations. Geometric actions: Platonic solids, Rubik’s Cube, Cayley graph. Cauchy’s theorem. Orbit-stabilizer Theorem, Sylow Theorems.

5. Rings, Fields, and Group Theory Definition and examples. Integral domains. Field of fractions. Maximal ideals. Quotient fields. Field extensions. Roots of polynomials. Fundamental theorem of Galois theory.

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Format

- This course features video lectures from the UIUC Spring 2016 course taught by Professor Chris Leininger and includes online lecture notes via Moodle. No additional textbook has to be purchased.

- These additional materials may be used for reference:
  Goodman, "Algebra: Abstract and Concrete" available at http://homepage.divms.uiowa.edu/~goodman/algebrabook.dir/algebrabook.html and Fraleigh, "A First Course in Abstract Algebra"

- Students must be able to print out assignments, write out solutions, then scan their written work and upload it to Moodle. Some elements of this course may require Flash Player. Please visit this link to ensure you have the latest version installed.

- This course requires multiple paper-based exams that must be taken with an approved proctor. Exams may be taken on campus with NetMath proctoring; for off-campus options see https://netmath.illinois.edu/offcampus. Off-campus proctors must be able to scan completed exams and email them to NetMath for grading, as well as mailing the paper exam back for archival purposes.