Math 416: Abstract Linear Algebra (3 credits)

Course Description

Math 416 is a rigorous, abstract treatment of linear algebra. Topics to be covered include vector spaces, linear transformations, eigenvalues and eigenvectors, diagonalizability, and inner product spaces. The course concludes with a brief introduction to the theory of canonical forms for matrices and linear transformations.

Credit is not given for both MATH 416 and either MATH 410 or MATH 415.

For more details see http://catalog.illinois.edu/courses-of-instruction/math/

Prerequisite: MATH 241; MATH 347 is recommended

Course Objectives

Students should leave the course with a basic understanding of the fundamental concepts of linear algebra and matrix theory, which is useful in many other areas including differential equations, physics and computer science. They should also gain improved ability at reading and writing mathematical arguments. Regular homework is an important aspect of the course.

Course Content

1. Basic Objects and Systems of Linear Equations
matrices, linear maps, examples of vector spaces and linear maps, row reduction and echelon form, augmented matrices and elementary row operations, (reduced) row echelon form and solutions of systems of linear equations, rank of a matrix

2. Vector Spaces
eight axioms of a vector space, subspaces, linear combinations, spanning sets, linear independence and bases, bases and dimension

3. Linear Transformations
nulll spaces and ranges, matrix representations of linear transformations, coordinate vectors, change of coordinates, the dimension theorem (or rank-nullity theorem), matrix multiplication and the composition of linear transformations, invertibility of matrices, inverse matrices

4. Elementary Row Operations
invertible matrices and elementary matrices, the rank of a matrix and linear independence of columns

5. Determinants
determinant of 2x2 matrices, determinant of nxn matrices, the linear property of determinant, determinant and invertibility, Cramer's rule

6. Diagonalization
eigenvalues and eigenvectors, eigenspaces, characteristic polynomials, diagonalizability, Markov chains, T-invariant subspaces, T-cyclic subspaces

7. Inner Product Space
orthogonality, Gram-Schmidt process, orthogonal complement, orthogonal projection, least square, adjoint operators, normal and self-adjoint operators, orthogonal and unitary matrices

8. Jordan Canonical Form
generalized eigenspace, Jordan canonical form, Jordan canonical bases

Format

- This is an online course featuring video lectures from the UIUC Fall 2016 course taught by Professor Ely Kerman.
- Text: Stephen H. Friedberg, Arnold J. Insel and Lawrence E. Spence. (2001). Linear Algebra (4th Edition). Prentice Hall.
- Students must be able view assignments online, write out solutions, then scan or take a photo of their written work and upload it to Moodle to meet set deadlines.
- This course requires multiple proctored exams.