Course Syllabus for **MATH 292** (Vector Calculus Supplement)

Course description: Course in multivariable calculus. Topics include gradient divergence and curl; line and surface integrals; and the theorems of Green, Stokes, and Gauss. Intended for transfer students whose multivariable calculus course did not include the integral theorems of vector calculus.

Vector Calculus Authors: Bill Davis, Horacio Porta, and Jerry Uhl ©2006-2010 Publisher: Making Math, a division of O'Reilly Media

Credit: 2 hours.

Prerequisite: Consent of instructor. Credit is not given for both MATH 292 and MATH 241.

Textbook: Please Visit - https://cas-ile.illinois.edu/

Syllabus:

VC.01 Vectors Point the Way

Mathematics:

Vectors. Adding and subtracting vectors. Tangent vectors, velocity vectors, and tangent lines. Length of a vector, dot product, and distance between two points. Perpendicular vectors. The push of one vector in the direction of another, and the formula

X•Y={|X|}{|Y|} Cos[b],

where b is the angle between X and Y.

Science and math experience:

Velocity and acceleration. Bouncing light rays off curves. Pursuit models. Laser gunnery. Planetary motion. Parabolic reflectors, spherical reflectors, and elliptical reflectors. Stealth technology.

VC.02 Perpendicularity

Mathematics:

The cross product X×Y of two 3D vectors. Lines and planes in 3D. Normal vectors for curved surfaces in 3D. Main unit normals, binormals.

Science and math experience:

Plotting tubes and corrugations. Using the normal vector to bounce light beams off surfaces. Kissing circles and curvature. Measurements with the cross product. Breaking acceleration vectors into normal and tangential components. Programming routers to cut specified pieces out of sheet steel. Plotting on planes. Serious 3D plots: tubes and ribbons.

VC.03 The Gradient

Mathematics:

The gradient and the chain rule. Level curves, level surfaces and the gradient as normal vector. The gradient points in the direction of greatest initial increase. Using linearizations to help to explain the chain rule. How to use the gradient for maximization and minimization. The total differential.

Science and math experience:
Estimating whether a given function has a global maximizer, minimizer, or both. Heat seeking missiles. Closest points, gradients and Lagrange's method. The Cobb-Douglas manufacturing model. Data Fit in two variables.

<u>VC.04 Vector Fields and their Trajectories</u>
Mathematics:
Flow of vector fields along curves. Flow of vector fields across curves. Differential equations and their associated vector fields.

Science and math experience:
Gradient fields try to flow toward maximizers. Looking for sinks (drains). Electric Fields. Normal components and flow across curves. Tangential components and flow along curves. Logistic harvesting. Plotting level curves. Gradient field versus Hamiltonian field.

<u>VC.05 Flow Measurements by Integral</u>
Mathematics:
Measuring flow across a curve with the integral
$\int_{tlow}^{thigh} (Field[x[t],y[t]].\{y'[t],-x'[t]\})dt$
Measuring flow along a curve with the integral
$\int_{tlow}^{thigh} (Field[x[t],y[t]].\{x'[t],y'[t]\})dt$
Measurements made with path integrals
$\int_{c} m[x,y]dx + n[x,y]dy$
Directed curves; path integrals, path independence and gradient fields. Recognizing gradient fields: the gradient test.

Science and math experience:
Path integrals: backwards and forwards. Water. Sources and sinks. Force fields. Work and how physicists measure it.

<u>VC.06 Sources, Sinks, Swirls, and Singularities</u>
Mathematics:
Gauss-Green formula. Using a 2D integral to measure flow across closed curves. Using a 2D integral to measure flow along closed curves. Using the divergence of a vector field to identify sources and sinks. Flow across a closed curve and flow along a closed curve. Measurements in the presence of singularities.

Science and math experience:
2D electric fields, dipole fields, and Gauss's law in physics.

$$\oint_c m[x,y]dx + n[x,y]dy$$

when

$$D[n[x,y],x] - D[m[x,y],y] = 0$$

The Laplacian $\frac{\partial^2 f[x,y]}{\partial x^2} + \frac{\partial^2 f[x,y]}{\partial y^2}$ and steady-state heat.

Maximum and minimum principle for functions $f[x,y]$ satisfying Laplace's equation

$$\frac{\partial^2 f[x,y]}{\partial x^2} + \frac{\partial^2 f[x,y]}{\partial y^2} = 0$$

Rotation and parallel flow.

VC.07 Transforming 2D Integral

Mathematics:

Going between uv-paper and xy-paper. Transforming 2D integrals: how you do it and why you do it. Linearizing the grids. Derivation of area conversion factor (Jacobian) via linearization.

Science and math experience:

How the plotting instructions reveal how to transform wicked 2D integrals into easy 2D integrals. Ribbons. More on flow measurements. Semi-log paper and log-log paper. What can happen at points at which the area conversion factor (Jacobian) is zero. What information the sign of the area conversion factor (Jacobian) reveals. Streamlines for flow out of an open pipe. Streamlines for airfoils.

VC.08 Transforming 3D Integrals

Mathematics:

3D integrals. Transforming wicked 3D integrals into easy 3D integrals. Volume measurements through transforming 3D integrals. Average value of a function.

Science and math experience:

How the plotting instructions reveal how to transform wicked 3D integrals into easy 3D integrals. Cylindrical coordinates. Centroids, and centers of mass. Cylinders, spheres, and tubes: plotting them and integrating on them. Switching the order of integration. Drilling and slicing spheres. The box product for measuring the volume of 3D parallelepipeds.

VC.06 Spherical Coordinates

Mathematics:

Spherical coordinates. Using spherical coordinates in 3D integration.

Science and math experience:

Using spherical coordinates to plot parts of spheres. Using spherical coordinates to plot cones and other surfaces. Earth-moon plots. Estimating the kill range of mobile lazer cones. Inserting planes between disjoint spheres. Spherical coordinate art. Measurements in four and five dimensions.

VC.10 3D Surface Measurements
Mathematics:
Divergence and Gauss's formula in 3D. Using the 3D divergence to identify sources and sinks in 3D vector fields. Surface integrals. Using surface integrals to measure flow across 3D surfaces.

Science and math experience:
3D electric fields and Coulomb's law. Gauss's 3D formula versus flow calculation via surface integrals. Using Gauss's formula to avoid a calculational nightmare: calculating flow across an oddball surface via calculating the flow across a substitute surface. Using Gauss's formula to take advantage of singularities: calculating flow across the skin of a solid region via calculating the flow across a substitute sphere. Flux of the electric field and Gauss's electric law in 3D. The 3D Laplacian

$\frac{\partial^2 f[x,y,z]}{\partial^2 x} + \frac{\partial^2 f[x,y,z]}{\partial^2 y} + \frac{\partial^2 f[x,y,z]}{\partial^2 z} = 0$ $\frac{\partial^2 f[x,y,z]}{\partial^2 x} + \frac{\partial^2 f[x,y,z]}{\partial^2 y} + \frac{\partial^2 f[x,y,z]}{\partial^2 z}$ and steady-state heat.

Maximum and minimum principle for functions f[x,y] satisfying Laplace's equation
$\frac{\partial^2 f[x,y,z]}{\partial^2 x} + \frac{\partial^2 f[x,y,z]}{\partial^2 y} + \frac{\partial^2 f[x,y,z]}{\partial^2 z} = 0.$

VC.11 3D Flow Along
Mathematics:
Measuring flow along a 3D curve via path integrals. The curl of a 3D vector field and what it measures. Orientation of surfaces. Stokes's formula in 3D. Path dependence and independence. The gradient test in 3D.

Science and math experience:
Force fields and the relation between flow-along measurements and work. Using 3D path integrals to construct functions with a given gradient.