Course Information

Course: MATH 446: Applied Complex Variables

Course Description:

MATH 446 is for students who desire a working knowledge of complex variables. The course covers the standard topics and gives an introduction to integration by residues, the argument principle, conformal maps, and potential fields. Students desiring a systematic development of the foundations of the subject should take MATH 448.

Course Objectives

The two main objectives are to develop theory that is useful in applications of complex variables and to introduce to applications of residues and conformal mapping. The former includes evaluating real improper integrals and locating zeros of functions and the latter includes solving boundary value problems in differential equations.

Course Content

1. Complex numbers
   Basic algebraic properties, absolute value of a complex number, triangle inequality, complex conjugate, polar form, Euler's formula, de Moivre's formula, argument, principal argument, nth roots of complex numbers, regions in the complex plane

2. Analytic functions
   Mapping by complex functions, limits, continuity and differentiability, Cauchy-Riemann equations, Cauchy-Riemann equations in polar form, analytic functions

3. Elementary functions
   Exponential and logarithmic functions, branches of logarithmic functions, principal branch of logarithmic functions, power functions for any complex exponents, principal branch of power functions, trigonometric functions, zeroes and singularities of trig. functions, hyperbolic functions

4. Integrals
   Contour integrals, upper bound of the absolute value of a contour integral, antiderivatives, Cauchy-Goursat theorem, Simply connected domain, Cauchy Integral Formula (CIF) and extended CIF, Liouville's Theorem and Fundamental Theorem of Algebra, Maximum Modulus Principle

5. Series

Convergence of series, power series, Taylor's series, Maclaurin series, Laurent series, sums of power series, integration and differentiation of power series, multiplication and division of power series

6. Residues and poles
   Isolated singularity, residues, Cauchy's residue theorem, residue at infinity, classification of isolated singularities (removable singularity, poles, essential singularity), zeros and poles

7. Applications of Resides
   Evaluation of real improper integrals, Jordan's lemma, real definite integrals of rational functions, logarithmic functions, power functions with arbitrary real exponents, rational functions of sine and cosine functions, Argument Principle, Rouche's Theorem

8. Mapping by elementary functions
   Linear transformation, Linear Fractional Transformation, Mappings of upper half-plane, mappings by exponential functions, trigonometric functions, square functions, square root functions

9. Conformal mapping
   Angle preservation, local inverse, harmonic conjugates, explanation of boundary value problems

10. Applications of conformal mapping
    Boundary value problems: Dirichlet problem, Neumann problem, and mixed problem

Format

- This is an online course featuring video lectures from the UIUC Summer 2019 course taught by Professor Joseph Miles.

- Required Text: James Ward Brown, Ruel V. Churchill. (2014). *Complex Variables and Applications*. (9th Edition). McGraw-Hill Higher Education.

- Students must be able to print out assignments, write out solutions, then scan their written work and upload it to Moodle.

- This course requires multiple paper-based exams that must be taken with an approved proctor. Exams may be taken on campus with NetMath proctoring; for off-campus options see https://netmath.illinois.edu/offcampus. Off-campus proctors must be able to scan

completed exams and email them to NetMath for grading, as well as mailing the paper exam back for archival purposes.