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Course Syllabus for **MATH 415** (Linear Transformations and Matrices)

Course description: Introductory course emphasizing techniques of linear algebra with applications to engineering; topics include matrix operations, determinants, linear equations, vector spaces, linear transformations, eigenvalues, and eigenvectors, inner products and norms, orthogonality, equilibrium, and linear dynamical systems.

Matrices and Geometry Authors: Bill Davis and Jerry Uhl ©2006-2010 Publisher: Making Math, a division of O'Reilly Media

Credit: 3 hours. (Note: 4 hours of credit requires approval of the instructor and department with completion of additional work of substance.)

Prerequisite: MATH 241 or equivalent.

Required Material: Please Visit - https://cas-ile.illinois.edu/

Syllabus:

MGM.00 PlotFest

Using *Mathematica* to plot in two and three dimensions with special attention to parametric plotting.

MGM.01 Perpendicular Frames in 2D and 3D

Vectors in 2D and vectors in 3D. Addition and subtraction of vectors. Dot product and Cross product. Aligning and hanging on perpendicular frames to plot tilted ellipses and ellipsoids. Right hand frames versus left hand frames. Resolution of vectors into perpendicular components. Planes and lines through the origin.

MGM.02 2D Matrix Action

Matrix multiplication. Hitting the unit circle with a matrix and observing the result through matrix action movies. Linearity of matrix multiplication. Taking a 2D perpendicular frame and using it to to plot tilted ellipses. Rotation matrices and right hand frames. Reflection matrices and left hand frames. Stretcher matrices. Why A.B is unlikely to be the same as B.A for given 2D matrices A and B. Inverse matrices.

MGM.03 Making 2D Matrices

Using two perpendicular frames and two stretch factors to make matrices whose hits have desired outcomes. Inverting matrices made this way. Making matrices whose hits stretch along a given perpendicular frame, making matrices whose hits reflect about a given line, making matrices whose hits project onto a given line. Ray tracing. Parabolic, spherical, elliptic and hyperbolic reflectors, stealth technology.

MGM.04 SVD Analysis of 2D Matrices

The SVD (Singular Value Decomposition) says that corresponding to any 2D matrix A are two perpendicular frames and two stretch factors that can be used to duplicate A. Using SVD stretch factors to recognize invertible matrices and then invert them. The determinant of a 2D matrix in terms of the SVD stretch factors. Why the determinant of Inverse[A] is the inverse of the determinant of A. Rank of a 2D matrix. Using 2D matrices to solve systems of linear equations. Eigenvalues and eigenvectors of 2D matrices.
Optional: Hand calculations involving Cramer's rule and Gaussian elimination.

MGM.05 3D Matrices

This lesson repeats the ideas of MGM.02, MGM.03 and MGM.04 in 3D.

MGM.06 Beyond 3D

The SVD (Singular Value Decomposition) says that corresponding to any arbitrary matrix A (square or non-square) are two perpendicular frames and a list of stretch factors that can be used to duplicate A. Rank of a matrix in terms of the SVD stretch factors. The meaning of full rank. Recognizing when a given system of n linear equations in k unknowns has:
a) exactly one solution (exactly determined).
b) many solutions (under determined)
c) no solution (over determined).
How to find find solutions of linear systems when they exist.
Using SVD to explicitly construct the PseudoInverse for getting best least squares solutions to over determined systems of linear equations.

MGM.07 Roundoff (Optional)

Creative rounding of matrices via the Singular Value Decomposition and image compression. Principal Component Analysis (PCA) of data via the Singular Value Decomposition. Ill-conditioned matrices: The trouble ill-conditioned matrices can cause and how to use the Singular Value Decomposition to recognize them.

MGM.08 Subspaces

Every set of vectors in nD spans a subspace of nD. Projecting onto a subspace of nD. Calculating the dimension of a subspace of nD. A set of k vectors in nD is linearly independent if it spans a k-dimensional subspace of nD. Traditional definitions of linear independence. Orthonormal sets. Gram Schmidt process. Alien plots coming from projections of highD surfaces onto 3D subspaces. Perpendicular complement of a subspace. Null spaces of matrices.

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MGM.09 Eigensense: Diagonalizable Matrices, Matrix Exponential, Matrix Powers and Dynamical Systems

Eigenvalues, eigenvectors, and using them to recognize diagonalizable matrices. Complex eigenvalues and eigenvectors. The matrix exponential for diagonalizable and non-diagonalizable matrices. Eigenvalues reveal long-term behavior of matrix exponentials and matrix powers. Using matrix exponentials and matrix powers to solve continuous dynamical systems (systems of linear differential equations) and discrete dynamical systems (systems of difference equations).

MGM.10 The Spectral Theorem for Symmetric Matrices and the Holy Grail of Matrix Theory

This is the main theoretical lesson. Discussion of the spectral theorem and its proof. Given an arbitrary matrix A, using the spectral theorem applied to Transpose[A].A to explain why every matrix has a singular value decomposition.
Using an orthonormal basis of eigenvectors of Transpose[A].A to read off:
a) an orthonormal basis of the column space R[A] of the matrix A
b) an orthonormal basis of the null space N[A] of the matrix A
c) an orthonormal basis of the row space of the matrix A
d) a construction of the PseudoInverse of the matrix A
Positive definite and positive semidefinite matrices. Quadratic forms. Grammian matrices.

MGM.11 Function Spaces

Functions as vectors. The root-mean-square distance between two functions on an interval. Weighted root-mean-square distance. The dot product of two functions. The component of one function in the direction of another. Orthogonal sets of functions: Sine systems, Cosine systems, Sin-Cosine systems, Legendre Polynomial system.
Sets of functions orthogonal with respect to a weight function. Chebyshev polynomials. Gram-Schmidt process. Fourier approximation and orthogonal functions. Fourier Sine approximation and the heat and wave equations. Using Fourier methods to bring the Dirac Delta function to life.