**Course Syllabus for MATH 286 (Introduction to Differential Equations)**

**Course description:** Intended for engineering students and others who require a working knowledge of differential equations; included are techniques and applications of ordinary differential equations and an introduction to partial differential equations.

Differential Equations by Bill Davis and Jerry Uhl © 2000 Publishers: Making Math

**Credit:** 4 hours.

**Prerequisite:** MATH 241. Credit is not given for both MATH 285 and any of MATH 284, MATH 286, MATH 441. (Counts for advanced hours in Liberal Arts and Sciences.)

**Textbook:** Please Visit - https://cas-ile.illinois.edu/

**Syllabus:**

**Part I: Transition from Calculus: Classical Theory of Differential Equations**

**DE.01** Transition from calculus: The Exponential diffeq $y'[t] + r y[t] = f[t]$  

- How to write down formulas for solutions of $y'[t] + r y[t] = 0$.
- How to use integrating factors to get formulas for solutions of $y'[t] + r y[t] = f[t]$.
- If $r > 0$, then all solutions of $y'[t] + r y[t] = f[t]$ go into the same steady state.
- Exponential models.
- The jump function `UnitStep[t-d]` and the impulse function `DiracDelta[t-d]`
- Impulse forcing the exponential diffeq with a Dirac Delta function; the physical meaning of the impulse force.
- The superposition principle.

**DE.02** Transition from Calculus: The Forced Oscillator diffeq $y''[t] + b y'[t] + c y[t] = f[t]$  

- The undamped unforced oscillator $y ''[t] + c y[t] = 0$
- The damped unforced oscillator $y ' ' [t] + b y ' [t] + c y[t] = 0$
- The damped forced oscillator $y ''[t] + b y ' [t] + c y[t] = f[t]$
- Steady state and transients for forced damped oscillators
- Resonance and beating
- Euler Identity

- The characteristic equation
- Using convolution integrals to try to get formulas solutions of the forced oscillator diffeq
- Resonance
- Forcing an oscillator with a Dirac Delta function; the physical meaning of the impulse hit
- Amplitude and frequency of unforced oscillators
- Underdamped, critically damped, and overdamped oscillators
Boundary value problems.

**DE.03 Transition from Calculus: Laplace Transform and Fourier Analysis**

The Laplace transform of a function \( y[t] \)

How to write down the Laplace transform of the solution of a forced oscillator diffeq

Solving forced oscillator diffeqs by inverting Laplace transforms

Fast Fourier point fit and Fourier integral fit

Combining Fourier fit and the Laplace transform to come up with good approximate formulas for periodically forced oscillators

Fourier analysis for detecting resonance

**Part II: Introduction to Modern Theory of Differential Equations**

**DE.04 Modern Diff Eq Issues**

Euler’s method of faking the plot of the solution of a differential equation and how it highlights the fundamental issue of diffeq

Reading a diffeq through flow plots

Solving diffeq’s numerically with Mathematica

Systems of interacting differential equations: the predator-prey model

Sensitive dependence on starter data

The drinking versus driving model

Population models and control; Logistic harvesting

Lanchester war model

**DE.05 Modern Diff Eq: First Order Differential Equations**

Reading an autonomous diffeq through phase lines

Autonomous diffeqs with parameters. Bifurcations and bifurcation points

Hand symbol manipulation: separating the variables

Population models and control

Using bifurcation plots to study E. Coli growing in a chemostat

Automatically controlled air conditioning

Getting there in infinite time versus getting there in finite time

**VC.06 Modern Diff Eq: Systems and Flows**

Flows and their trajectories as pairs of solutions of a system of differential equations

Flow analysis of the unforced linear oscillator differential equation by converting it to a system of two first order differential equations

Equilibrium points

Damped oscillators, undamped oscillators and van der Pol’s nonlinear oscillator

Linear systems and graphical meaning of eigenvectors of the coefficient matrix

Pursuit models

Boundary value problems: Shooting for a specified outcome
DE.07 Modern Diff Eq: Eigenvectors and Eigenvalues for Linear Systems

Eigenvectors of the coefficient matrix point in the directions of strongest inward and/or outward flow
Eigenvalues of the coefficient matrix indicate relative strengths of inward and/or outward flow
Eigenvalue-trajectory analysis to predict swirl in, swirl out or no swirl at all
Stability and instability
Reservoir Models for drug metabolism
Linear systems in life science, chemistry and electrical engineering
Higher dimensional linear systems.

DE.08 Modern Diff Eq: Linearization of Nonlinear Systems

Using the Jacobian to approximate a nonlinear diffeq system by linearizing at equilibrium points
Attractors and repellers: Lyapunov's rules for detecting them via analysis of the eigenvalues of the Jacobian
The pendulum oscillator: damped and undamped
When linearizations can be trusted and when they shouldn't be trusted
Linearization of pendulum oscillators: Using linearization to estimate the amplitude and frequency of a pendulum oscillator
Energy and the undamped pendulum oscillator
The Van der Pol oscillator
Gradient and Hamiltonian systems
Lorenz’s chaotic oscillator.

Part III: Partial Diff Eq: Heat and Wave Equations

DE.09 Heat Equation and Wave Equation
Rigging \( f(t) \) on \([0, L]\) to get a pure sine fast Fourier fit of \( f(t) \) on \([0, L]\)
Fourier Sine fit for solving the heat equation
Fourier Sine fit for solving the wave equation
Solving the heat and the wave equations in the case that initial data are given by a data list.