

**NetMath Online Math Courses, University of Illinois**Course Syllabus for **MATH 285** (Introduction to Differential Equations)**Course description:** Intended for engineering students and others who require a working knowledge of differential equations; included are techniques and applications of ordinary differential equations and an introduction to partial differential equations.

Differential Equations Authors: Bill Davis and Jerry Uhl ©2006-2010 Publisher: Making Math, a division of O'Reilly Media

Credit: 3 hours.**Prerequisite:** MATH 241. Credit is not given for both MATH 285 and any of MATH 284, MATH 286, MATH 441. (Counts for advanced hours in Liberal Arts and Sciences.)**Textbook:** Please Visit - <https://cas-ile.illinois.edu/>**Syllabus:****Part I: Transition from Calculus: Classical Theory of Differential Equations****DE.01** Transition from calculus: The Exponential diffeq $y'[t] + r y[t] = f[t]$ How to write down formulas for solutions of $y'[t] + r y[t] = 0$.How to use integrating factors to get formulas for solutions of $y'[t] + r y[t] = f[t]$.If $r > 0$, then all solutions of $y'[t] + r y[t] = f[t]$ go into the same steady state.

Exponential models.

The jump function $\text{UnitStep}[t-d]$ and the impulse function $\text{DiracDelta}[t-d]$

Impulse forcing the exponential diffeq with a Dirac Delta function; the physical meaning of the impulse force.

The superposition principle.

DE.02 Transition from Calculus: The Forced Oscillator diffeq $y''[t] + b y'[t] + c y[t] = f[t]$ The undamped unforced oscillator $y''[t] + c y[t] = 0$ The damped unforced oscillator $y''[t] + b y'[t] + c y[t] = 0$ The damped forced oscillator $y''[t] + b y'[t] + c y[t] = f[t]$

Steady state and transients for forced damped oscillators

Resonance and beating

Euler Identity

The characteristic equation

Using convolution integrals to try to get formulas solutions of the forced oscillator diffeq

Resonance

Forcing an oscillator with a Dirac Delta function ; the physical meaning of the impulse hit

Amplitude and frequency of unforced oscillators

Underdamped, critically damped, and overdamped oscillators
Boundary value problems.

DE.03 Transition from Calculus: Laplace Transform and Fourier Analysis

The Laplace transform of a function $y[t]$
How to write down the Laplace transform of the solution of a forced oscillator diffeq
Solving forced oscillator diffeqs by inverting Laplace transforms
Fast Fourier point fit and Fourier integral fit
Combining Fourier fit and the Laplace transform to come up with good approximate formulas for periodically forced oscillators
Fourier analysis for detecting resonance

Part II: Introduction to Modern Theory of Differential Equations

DE.04 Modern Diff Eq Issues

Euler's method of faking the plot of the solution of a differential equation and how it highlights the fundamental issue of diffeq
Reading a diffeq through flow plots
Solving diffeq's numerically with Mathematica
Systems of interacting differential equations: the predator-prey model
Sensitive dependence on starter data
The drinking versus driving model
Population models and control ; Logistic harvesting
Lanchester war model

DE.05 Modern Diff Eq: First Order Differential Equations

Reading an autonomous diffeq through phase lines
Autonomous diffeqs with parameters. Bifurcations and bifurcation points
Hand symbol manipulation: separating the variables
Population models and control
Using bifurcation plots to study E. Coli growing in a chemostat
Automatically controlled air conditioning
Getting there in infinite time versus getting there in finite time

VC.06 Modern Diff Eq: Systems and Flows

Flows and their trajectories as pairs of solutions of a system of differential equations
Flow analysis of the unforced linear oscillator differential equation by converting it to a system of two first order differential equations
Equilibrium points
Damped oscillators, undamped oscillators and van der Pol's nonlinear oscillator
Linear systems and graphical meaning of eigenvectors of the coefficient matrix
Pursuit models

Boundary value problems: Shooting for a specified outcome

DE.07 Modern Diff Eq: Eigenvectors and Eigenvalues for Linear Systems

Eigenvectors of the coefficient matrix point in the directions of strongest inward and/or outward flow

Eigenvalues of the coefficient matrix indicate relative strengths of inward and/or outward flow

Eigenvalue-trajectory analysis to predict swirl in, swirl out or no swirl at all

Stability and instability

Reservoir Models for drug metabolization

Linear systems in life science, chemistry and electrical engineering

Higher dimensional linear systems.

VC.08 Modern Diff Eq: Linearization of Nonlinear Systems

Using the Jacobian to approximate a nonlinear diffeq system by linearizing at equilibrium points

Attractors and repellers: Lyapunov's rules for detecting them via analysis of the eigenvalues of the Jacobian

The pendulum oscillator: damped and undamped

When linearizations can be trusted and when they shouldn't be trusted

Linearization of pendulum oscillators: Using linearization to estimate the amplitude and frequency of a pendulum oscillator

Energy and the undamped pendulum oscillator

The Van der Pol oscillator

Gradient and Hamiltonian systems

Lorenz's chaotic oscillator.

Part III: Partial Diff Eq: Heat and Wave Equations

DE.09 Heat Equation and Wave Equation

Rigging $f[t]$ on $[0, L]$ to get a pure sine fast Fourier fit of $f[t]$ on $[0, L]$

Fourier Sine fit for solving the heat equation

Fourier Sine fit for solving the wave equation

Solving the heat and the wave equations in the case that initial data are given by a data list.