NetMath Online Math Courses, University of Illinois

Course Syllabus for **MATH 234** (Calculus for Business I)

Course description: Introduction to the concept of functions and the basic ideas of calculus.

Credit: 4 hours.

Prerequisite: MATH 012.

Required Material: Please Visit - https://cas-ile.illinois.edu/

Syllabus:

Assignment	Week	Problems
Assignment 0	Week 0	Go through Feel of Mathematica
Assignment 1	Week 1	Growth
Assignment 2	Week 2	The Exponential and Log Functions
Assignment 3	Week 3	Instantaneous Growth
Assignment 4	Week 4	Rules of the Derivative
Assignment 5	Week 5	Using the Tools
Assignment 6	Week 6	Differential Equations
Assignment 7	Week 7	The Race Track Principle
Assignment 8	Week 8	More Differential Equations
Assignment 9	Week 9	Parametric Plotting
Assignment 10	Week 10	Measuring Area
Assignment 11	Week 10	The Fundamental Formula
Assignment 12	Week 11	Parametric Plotting
Assignment 13	Week 12	Measurements with Integrals
Assignment 14	Week 13	Transforming Integrals
Assignment 15	Week 14	Integrals and Gauss-Green Formula
Assignment 16	Week 15	More Tools and Measurements
Assignment 17	Week 16	Traditional Pat Procedures

1. Growth

1.01 Growth
Mathematics:
Line functions and polynomials. Interpolation of data. Compromise lines through data. Dominant terms in the global scale.

Science and math experience:

Reading plots. Linear models. Drinking and driving. Japanese economy cars versus American big cars. Data analysis and interpolation. Data analysis of U.S. national debt and U.S. population in historical context, including plots of yearly growth and the effect of immigration on the growth of the U.S. population. Cigarette smoking and lung cancer correlation. Global scale of quotients of functions studied by looking at dominant terms in the numerators and denominators.

1.02 Natural Logs and Exponentials

Mathematics:

How to write exponential and logarithm functions in terms of the natural base e. While line functions post a constant growth rate, exponential functions post a constant percentage growth rate. How to construct a function with a prescribed percentage growth rate.

Science and math experience:

Recognition of exponential data, exponential data fit, carbon dating, credit cards, compound interest, effective interest rates, financial planning, decay of cocaine in the blood, underwater illumination, inflation.

1.03 Instantaneous Growth Rates

Mathematics:

The instantaneous growth rate $f'[x]$ as the limiting case of the average growth rates $\frac{f[x+h]-f[x]}{h}$. What it means when $f'[x]$ is positive or negative. Calculation of $f'[x]$ for functions $f[x]$ like x^k, $Sin[x]$, $Cos[x]$, e^x and $Log[x]$. Why $Log[x]$ is the natural logarithm and why e is the natural base for exponentials. Max-min.

Science and math experience:

Relating the plots of $f[x]$ and $f'[x]$. Using a plot of $f'[x]$ to predict the plot of $f[x]$. Visualizing the limiting process by plotting $f'[x]$ and $\frac{f[x+h]-f[x]}{h}$ on the same axes and seeing the plots coalesce as h closes in on 0. Spread of disease model. Instantaneous growth rates in context.

1.04 Rules of the Derivative

Mathematics:

The derivative as the instantaneous growth rate. Chain rule. Product rule as a consequence of the chain rule. Instantaneous percentage growth rate $\frac{100 f'[x]}{f[x]}$ of a function $f[x]$.

Science and math experience:

Another look at why exponential growth dominates power growth and why power growth dominates logarithmic growth. Logistic model of animal growth. The idea of linear

dimension and using it to convert a model of animal height as a function of age to a model of animal weight as a function of age. Learning why the adolescent growth spurt is probably a mathematical fact instead of a biological accident. Compound interest. Making functions with prescribed instantaneous percentage growth rate.

1.05 Using the Tools
Mathematics:
What it means when $f'[x]>0$ for $x=a$.
Why $f[x]$ is not as big (or small) as it can be at $x=a$ unless $f'[a]=0$.

Science and math experience:
Why a good representative plot of a given function $f[x]$ usually includes all x's at which $f'[x]=0$. Max-Min in one or two variables. Using the derivative to get best least squares fit of data by smooth curves. Fitting of space shuttle O-ring failure data as a function of temperature and using the result to explain why the Challenger disaster should have been predicted in advance. Data fit by lines and by Sine and Cosine waves. Optimal speed for salmon swimming up a river. Designing the least cost box to hold a given volume. Analysis of an oil slick at sea. How tall is the dog when it is growing the fastest? Analysis of what happens to $x^t e^{-x}$ as x advances from 0 to $+\infty$.

1.06 The Differential Equations of Calculus
Mathematics:
The three differential equations

$\quad y'[x]=r\,y[x]$,

$\quad y'[x]=r\,y[x]\,(1-\dfrac{y[x]}{b})$

$\quad y'[x]=r\,y[x]+b$

and their solutions.
The meaning of the parameters r and b in the three differential equations. Why it's often a good idea to view logistic growth as toned down exponential growth.

Science and math experience:
Models based on these differential equations. Why radioactive decay is modeled by the differential equation $y'[x]=r\,y[x]$. Logistic versus exponential growth. Biological principles behind carbon dating. Growth of U.S. and world populations: Malthusian versus logistic models. Calculation of interest payments resulting from buying a car on time. Managing an inheritance. Wal-mart sales. Pollution elimination, data analysis, speculating on why dogs and humans grow faster after their birth than they are at the instant of their birth, but horses grow fastest at the instant of their birth. Newton's law of cooling. Pressure altimeters.

1.07 The Race Track Principle
Mathematics:
The race track principles:
If $f[a] = g[a]$ and $f'[x] \leq g'[x]$ for $x \geq a$, then $f[x] \leq$ for $x \geq a$.

If $f[a] = g[a]$ and $f'[x]$ is approximately equal to $g'[x]$ for $x \geq a$, then $f[x]$ is approximately equal to $g[x]$ for $x \geq a$.

If $f[a] = g[a]$ and $f'[x] = g'[x]$ for $x \geq a$, then $f[x] = g[x]$ for $x \geq a$.

Euler's method of faking the plot of a function with a given derivative explained in terms of the race track principles.

Euler's method of faking the plot of the solution of a differential equation explained in terms of the race track principles.

Science and math experience:

Using the race track principle to explain why, as x advances from 0, the plots of solutions of

$$y'[x] = r\, y[x] \text{ and } y'[x] = r\, y[x]\, (1 - \frac{y[x]}{b})$$

will run close together in the case that $y[0]$ is small relative to b. Why $Sin[x] \leq x$ for $x \geq 0$ and related inequalities. Estimating how many accurate decimals of x are needed to get k accurate decimals of $f[x]$. The error function. Calculating accurate values of $Log[x]$ and e^x.

1.08 More Differential Equation
Mathematics:

Plots of numerical approximations to solutions of first order differential equations. Qualitative analysis of first order differential equations and systems of first order differential equations.

Science and math experience:

Analysis of the predator-prey model. Cycles in the predator-prey model. Drinking and driving model. Variable interest rates. Michaelis-Menten Drug equation. War games based on Lanchester war model, including a simulation of the Battle of Iwo Jima. Harvesting in the logistic model. SIR epidemic model. The idea of chaos.

2. Accumulation

2.01 Integrals for Measuring Area
Mathematics:

Integrals defined as area measurement as done in E. Artin's MAA notes written in the 1950's. Approximations by trapezoids.

Science and math experience:

Integrals of functions given by data lists. Using known area formulas for triangles, trapezoids and circles to calculate integrals. Odd functions. Trying to break the code of the integral by taking selected functions g[x], putting

$$f[x] = \int_a^x g(t)\, \mathrm{dt}$$

and plotting

$$\frac{f[x+h] - f[x]}{h} \text{ and } g[x].$$

on the same axes for small h's. Plotting $f[x] = \int_a^x Cos(t)dt$ and guessing a formula for $f[x]$.

Plotting $f[x] = \int_a^x Sin(t)dt$ and guessing a formula for $f[x]$. Estimating the acreage of farm field bordered by a river.

2.02 The Fundamental Formula
Mathematics:

If $f[t]$ is given by $f[x] = \int_a^x g(t)dt$ then $f'[x] = g[x]$.

The fundamental formula $f[x]\text{-}f[a] = \int_a^x f(t)dt$.

Science and math experience:
Relating distance, velocity and acceleration through the fundamental formula. Getting the feel of the fundamental formula by using it to calculate integrals by hand. Relating

$$\int_a^x g(t)dt$$

to the solution of the differential equation
   $y'[x] = g[x]$ with y[a]=0.
Very brief look at the "indefinite integral" $\int g(t)dt$

Measuring area between curves. The error function, *erf*[x], and other functions defined by integrals. Measurements of acculumulated growth. Coloring ceramic tiles.

2.03 3D Measurements
Mathematics:
Measurements based on slicing and accumulating: area and volume; density and mass. Measurements based on approximating and measuring: arc length. Measurements based on the fundamental formula: accumulated growth.

Science and math experience:
Volumes of solids with no special emphasis on solids of rotation. Volume measurements of curved tubes and horns. Eyeball and precise estimates of curve lengths. Filling water tanks. Harvesting corn. Voltage drop. Another look at linear dimension. Work. Present value of a profit-making scheme. Catfish harvesting. Designing an 8 fluid ounce logarithmic champagne glass.

2.04 Transforming Integrals
Mathematics:
Using the chain rule and the fundamental formula to see why
$$\int_a^b f'[u[x]]u'[x]dx = \int_{u[a]}^{u[b]} f'[u]du$$

and using this fact to transform one integral into another. Measuring area under curves given parametrically. Bell shaped curves and Gauss' normal probability law; mean and standard deviation.

Science and math experience:
Study of the error function, erf[x]. Using tranformations to explain *Mathematica* output. Polar plots and area measurements. Using transformations to explain the meaning of standard deviation in Gauss's normal law. Expected life of light bulbs and how long to set the guarantee on them. Using Gauss's normal law to help to program coin-operated coffee machines. IQ test results. Using Gauss' normal law to organize SAT scores into quartiles and deciles. Comparison of 1967 and 1987 SAT scores. "Grading on the curve."

2.05 2D Integrals and the Gauss-Green Formula
Mathematics:
Meaning of the plot of z=f[x,y]. The 2D integral
$$\int_a^b \int_c^d f[x,y] dx dy$$

as a volume measurement via slicing and acculumulating. Gauss-Green formula (Green's theorem) as a way of calculating a double integral numerically as a single integral.

Science and math experience:
Volume and area measurements with 2D integrals. Area and volume measurements via the Gauss-Green formula. Average value and centroids. Calculation strategies. Plotting and measuring. Gauss's normal law in 2D and using it, as done in the Pentagon, to decide how many bombs to drop on a target.

2.06 More Tools and Measurements
Mathematics:
Separating the variables and integrating to get formulas for the solutions of some differential equations. Integration by parts. Complex numbers and the complex exponential
$E^{I\,t}$=Cos[t] + I Sin[t].

Science and math experience:
Formulas for the solutions of the differential equations involved in the chemical model and the spread of infection model. Hyperbolic functions and their relation to trigonometric functions. Using the complex exponential to help to understand the *Mathematica* output from the Solve instruction. Gamma function. Integration by parts and integration by iteration. Error propagation in forward iteration. Error reduction by backwards iteration.

2.06 Traditional Pat Integration Procedures
Mathematics:
Undetermined coefficients. Complex numbers and partial fractions. Wild card substitutions with the help of a trigonometric, hyperbolic or ad hoc function. Integration by parts.

Science and math experience:
Not much, although the experience gained from trying the method of undetermined coefficients is good experience in setting up and solving systems of linear equations.