NetMath Online Math Courses, University of Illinois

Course Syllabus for **MATH 231** (Calculus II)

Course description: Second course in calculus and analytic geometry: techniques of integration, conic sections, polar coordinates, and infinite series.

Credit: 3 hours.

Prerequisite: MATH 220. Credit is not given for both MATH 230 and MATH 231.

Required Material: Please Visit - https://cas-ile.illinois.edu/

Syllabus:

3. Approximation

3.01 Splines
Mathematics:
Remarkable plots explained by order of contact. Splining for smoothness at the knots.

Science and math experience:
Experiments geared at discovering that the smoother the transition from one curve to another at a knot, the better both curves approximate each other near the knot. Splining functions and polynomials. Splines in road design. Landing an airplane. The natural cubic spline. Order of contact for derivatives and integrals.

3.02 Expansions in powers of x
Mathematics:
The expansion of a function f[x] in powers of x as a file of polynomials with higher and higher orders of contact with f[x] at x=0. The expansions every literate calculus person knows:

$1/(1 - x)$, E^x, Sin[x], and Cos[x].
Converting known expansions to others via change of variable.
Expansions for approximations.

Science and math experience:
Experiments geared toward discovering that using more and more of the expansion results in better and better approximation. Halley's way of calculating accurate decimals of π. Expansions by substitution. Expansions by differentiation. Expansions by integration. Recognition of expansions. Expansions that satisfy a priori error bounds.

3.03 Using Expansions

Mathematics:
The expansion of a function f[x] in powers of (x-b) as a file of polynomials with higher and higher orders of contact with f[x] at x=b. Netwon's method. Multiplying and dividing expansions. Using expansions to help calculate limits at a point. Expansions and the complex exponential function. Using expansions to help get precise estimates of some integrals.

Science and math experience:
Centering expansions for good approximation. Newton's method for root finding. Successes and failures of Newton's method. Using the complex exponential to generate trigonometric identities. Comparing reflecting properties of spherical mirrors and the reflecting properties of parabolic mirrors. Using expansions to see why spherical mirrors have limited ability to concentrate light rays. Behavior of expansions very close to 0. Behavior of expansions far away from 0.

3.04 Taylor's Formula
Mathematics:
Taylor's formula for expansions in powers of (x-b).

Science and math experience:
Euler, Midpoint and Runge-Kutta approximations of f[x] given f′[x]. Experiments comparing the quality of Midpoint and Runge-Kutta approximations. Adaption of Euler, Midpoint and Runge-Kutta approximations to approximating the plots of the differential equation y′[x]=f[x,y[x]], with y[a] given. Taylor's formula in reverse. L'Hospital's rule by dividing the leading term of the expansion of the denominator into the leading term of the expansion of the numerator. Centering the expansion for best approximation. Experiments comparing the derivative of the expansion and the expansion of the derivative.

3.05 Barriers to Convergence
Mathematics:
Barriers and complex singularities. The convergence interval of an expansion as the interval between the barriers. Why some functions such as $1/(1 + x^2)$ have convergence barriers and others such as E^x and Sin[x] do not. Why functions such as $x^{(1/3)}$ and Log[x] do not have expansions in powers of x but do have expansions in powers of (x-b) for b>0. Why the convergence intervals for f[x], f′[x] and $\int_a^x f[t]dt$ are the same.

Science and math experience:
Shortcuts based on the expansion of 1/(1 - x) in powers of x. Using the expansion of 1/(1 - x) in powers of x for drug dosing. Infinite sums of numbers resulting from expansions. Barriers resulting from splines. Infinite sums and decimals. Experiments relating expansions in powers of x to interpolating polynomials. Runge's disaster.

3.06 Power Series
Mathematics:

Functions defined by a power series. Functions defined by power series via differential equations. The power series convergence principle, which says that if for some positive number r the infinite list

   $\{a_0, a_1\,r, a_2\,r^2, a_3\,r^3, ..., a_n\,r^n, ...\}$

is bounded, then the power series

   $a_0 + a_1\,r + a_2\,r^2 + a_3\,r^3 + ...+ a_n\,r^n + ...$

converges for -r<x<r.

Science and math experience:
Experiments in trying to plot functions defined by power series. Experiments in plotting a function defined by a power series via a differential equation versus plotting the same function directly through *Mathematica*'s numerical differential equation solver. The ratio test for power series as a consequence of the power series convergence principle.
The functions E^x, Sin[x] and Cos[x] from the viewpoint of power series. Experiments in truncation of power series. The Airy function as a function defined by a power series.

2.04 Transforming Integrals
Mathematics:
Using the chain rule and the fundamental formula to see why

$$\int_a^b f'[u[x]]u'[x]dx = \int_{u[a]}^{u[b]} f'[u]du$$

and using this fact to transform one integral into another. Measuring area under curves given parametrically. Bell shaped curves and Gauss' normal probability law; mean and standard deviation.

Science and math experience:
Study of the error function, erf[x]. Using transformations to explain *Mathematica* output. Polar plots and area measurements. Using transformations to explain the meaning of standard deviation in Gauss's normal law. Expected life of light bulbs and how long to set the guarantee on them. Using Gauss's normal law to help to program coin-operated coffee machines. IQ test results. Using Gauss' normal law to organize SAT scores into quartiles and deciles. Comparison of 1967 and 1987 SAT scores. "Grading on the curve."

2.05 2D Integrals and the Gauss-Green Formula
Mathematics:
Meaning of the plot of z=f[x,y]. The 2D integral

$$\int_a^b \int_c^d f[x,y]dxdy$$

as a volume measurement via slicing and acculumulating. Gauss-Green formula (Green's theorem) as a way of calculating a double integral numerically as a single integral.

Science and math experience:
Volume and area measurements with 2D integrals. Area and volume measurements via the Gauss-Green formula. Average value and centroids. Calculation strategies. Plotting and measuring. Gauss's normal law in 2D and using it, as done in the Pentagon, to decide how many bombs to drop on a target.

2.06 More Tools and Measurements

Mathematics:

Separating the variables and integrating to get formulas for the solutions of some differential equations. Integration by parts. Complex numbers and the complex exponential

$E^{I\,t}=\text{Cos}[t] + I\,\text{Sin}[t]$.

Science and math experience:

Formulas for the solutions of the differential equations involved in the chemical model and the spread of infection model. Hyperbolic functions and their relation to trigonometric functions. Using the complex exponential to help to understand the *Mathematica* output from the Solve instruction. Gamma function. Integration by parts and integration by iteration. Error propagation in forward iteration. Error reduction by backwards iteration.

2.06 Traditional Pat Integration Procedures

Mathematics:

Undetermined coefficients. Complex numbers and partial fractions. Wild card substitutions with the help of a trigonometric, hyperbolic or ad hoc function. Integration by parts.

Science and math experience:

Not much, although the experience gained from trying the method of undetermined coefficients is good experience in setting up and solving systems of linear equations.