



Math 453: Number Theory

Offered through **NetMath**[®]

Academic Year 2026-2027 Term, 16-week course

Instructor Information

Instructor – Jennifer Lansing

- Contact information: math453@netmath.illinois.edu
- Office hours are announced weekly via Canvas.

Course Information

Credit Hours

3 credit hours

Course Description

Basic introduction to the theory of numbers. Core topics include divisibility, primes and factorization, congruences, arithmetic functions, quadratic residues and quadratic reciprocity, primitive roots and orders. Additional topics covered at the discretion of the instructor include sums of squares, Diophantine equations, continued fractions, Farey fractions, recurrences, and applications to primality testing and cryptography.

Prerequisite

MATH 241; MATH 314 or MATH 347

Course Objectives:

Students should leave the course with an understanding of fundamental concepts of elementary number theory. They should also gain improved ability at reading and writing mathematical arguments. Because the subject of this course is defined more by the motivating problems than the techniques used to solve them, students should become comfortable experimenting with various approaches to problem solving and should gain experience choosing appropriate tools.

Course Content:

- Divisibility and Factorization
 - Divisibility: Definition, properties, division algorithm, greatest integer function
 - Primes: Definition, Euclid's Theorem, Prime Number Theorem (statement only), Goldbach and Twin Primes conjectures, Fermat primes, Mersenne primes
 - The greatest common divisor: Definition, properties, Euclid's algorithm, linear combinations and the gcd
 - The least common multiple: Definition and properties,
 - The Fundamental Theorem of Arithmetic: Euclid's Lemma, canonical prime factorization, divisibility, gcd, and lcm in terms of prime factorizations
 - Primes in arithmetic progressions: Dirichlet's Theorem on primes in arithmetic progressions (statement only)
- Congruences
 - Definitions and basic properties, residue classes, complete residue systems, reduced residue systems
 - Linear congruences in one variable, Euclid's algorithm
 - Simultaneous linear congruences, Chinese Remainder Theorem
 - Wilson's Theorem
 - Fermat's "Little" Theorem, pseudoprimes
 - Euler's Theorem

- Arithmetic Functions
 - Arithmetic function, multiplicative functions: definitions and basic examples
 - The Moebius function, Moebius inversion formula
 - The Euler phi function
 - The number-of-divisors and sum-of-divisors functions
 - Perfect numbers, characterization of even perfect numbers
- Quadratic Residues
 - Quadratic residues and nonresidues
 - The Legendre symbol: Definition and basic properties, Euler's Criterion, Gauss' Lemma
 - The law of quadratic reciprocity
- Primitive Roots
 - The order of an integer
 - Primitive roots: Definition and properties
 - The Primitive Root Theorem: Characterization of integers for which a primitive root exists
- Additional topics
 - Sums of squares
 - Pythagorean triples
 - Continued fractions and rational approximations

Course Structure:

This course is fully online and asynchronous. Students have 16 weeks from their course start date to complete the course. The course is self-paced, and students may complete their coursework prior to the assigned end date. However, students are encouraged to follow the guided structure provided by the instructor with suggested deadlines for assignments and exams. As this is a 3-credit hour course, students are expected to dedicate 9 hours per week to learning course content.

There are 11 units in this course. Each unit begins with an overview, and the learning goals you are expected to achieve. These goals should guide your study through the unit. Every unit consists of one homework assignment, lectures, readings and additional exercises to support these goals. They are designed with the same structure and components unless otherwise specified. There are approximately 3-4 units covered on each midterm test. It is recommended that you pace yourself by completing one unit a week.

You should complete the homework using the concepts introduced in the lectures and book. For example, while there is a lot of overlap between number theory and abstract algebra, and ideas from abstract algebra can be used to prove results in number theory, please only use the methods from the course lectures. Do not use later topics to prove the beginning easier topics, and do not bring in theorems or results that are not covered in the course book.

Exams:

Information	Exam 1	Exam 2	Exam 3	Final Exam
Coverage:	Lecture 1-13	Lecture 14-26	Lecture 27-39	Comprehensive
Time Limit:	60 Minutes	60 Minutes	60 Minutes	3 Hours

All exams will be administered online, free of charge to students. Exams are administered using ProctorU Record+. [More information on taking exams.](#)

Calculators are not permitted or required for NetMath exams.

The use of outside sources is prohibited.

Refer to the Academic Integrity policies below in the syllabus for more details.

Scratch paper is allowed during exams.

Exams must be taken in course order and students should not take more than 1 exam within a 24-hour period.

Grading:

Grading Breakdown:

Component	Weight
Homework	25%
Exam 1	15%
Exam 2	15%
Exam 3	15%
Final Exam	30%

Letter Grade Scale:

Percentage	Letter Grade
97 - 100	A+
93 - 96	A
90 - 92	A-
87 - 89	B+
83 - 86	B
80 - 82	B-
77 - 79	C+
73 - 76	C
70 - 72	C-
67 - 69	D+
63 - 66	D
60 - 62	D-
59 and below	F

Final grades are rounded to the nearest integer before applying the scale above.

Grades are not curved.

Once you complete your exam, you will receive a 0 on any homework leading up to that exam that has not been submitted. Under special circumstances, students may request that they be allowed to submit these assignments. This is left to the discretion of the course instructor.

Students are encouraged to follow the guided deadlines in their course. However, as our courses are self-paced, students may occasionally submit several assignments in a short period of time. Please note that instructors are only expected to grade one exam per five-business-day week, and instructors are only expected to grade up to two submitted assignments per five-business-day week. Under special circumstances, students may request that additional assignments be graded. This is left to the discretion of the course instructor.

Due to the rolling enrollment structure of the NetMath program, instructor workloads vary throughout the year. During peak periods, response times and grading turnaround may be longer than usual. Students should expect significant delays in grading and instructor responses between June and August.

Communication:

You may reach out to the instructor via email or attend office hours.

If you experience consistent and significant delays in reaching your instructor, please contact instruction@netmath.illinois.edu with your concerns.

If you have non-course specific questions, you may reach out to the NetMath office. It is recommended to [review the NetMath website](#) first as it often has the answers you need.

General Inquiries: netmath@illinois.edu

Proctoring Questions: proctor@netmath.illinois.edu

Technology Questions: tech-staff@netmath.illinois.edu

Main Office Phone: [\(217\) 265-0439](tel:(217)265-0439)

Phone hours: Monday - Friday, 9am - 12pm & 1pm - 4pm CST

Textbook:

James K. Strayer. (2002). *Elementary Number Theory*. (Reissue Edition). Waveland Press, Inc.

Technical Requirements:

Students are expected to have a device with an up-to-date operating system to access their course materials, reliable internet access, and must meet the requirements set by ProctorU for exam proctoring. [NetMath IT - Frequently Asked Questions](#).

Other Information:

This is an online course featuring video lectures from the UIUC Spring 2018 course taught by Professor Bruce Berndt.

Policies:

Accommodations:

The University of Illinois Champaign-Urbana, is committed to ensuring that all students, including students with disabilities, do not experience barriers to learning and participating fully in class. Students who have a letter of accommodation from DRES are advised to share that with their instructor and NetMath staff at instruction@netmath.illinois.edu as soon as possible to ensure accommodation needs can be discussed and met.

Students enrolled in a degree program at another institution may refer to their home institution's student services or disability office for accommodations. Current, non-expired documentation from other institutions listing the specifics of an accommodation is acceptable and should be sent to instruction@netmath.illinois.edu.

To obtain disability-related academic accommodations, disabled students must contact Disability Resources and Educational Services (DRES) as soon as possible. To contact DRES, please visit 1207 S. Oak Street, Champaign, call [\(217\) 333-1970](tel:(217)333-1970), email disability@illinois.edu, or [visit the DRES website](#).

Academic Integrity:

The University of Illinois Urbana-Champaign [Student Code](https://studentcode.illinois.edu/) (https://studentcode.illinois.edu/) should also be considered as a part of this syllabus. Students should pay particular attention to Article 1, Part 4: Academic Integrity.

Academic dishonesty may result in a failing grade. Every student is expected to review and abide by the [Academic Integrity Policy](https://studentcode.illinois.edu/article1/part4/1-401/) (https://studentcode.illinois.edu/article1/part4/1-401/). Ignorance is not an excuse for any academic dishonesty. It is your responsibility to read this policy to avoid any misunderstanding. Do not hesitate to ask the instructor(s) if you are ever in doubt about what constitutes plagiarism, cheating, or any other breach of academic integrity.

AI Large Language Models (such as ChatGPT, Claude, Gemini, etc.) are prohibited during exams. Using these during the exam will be considered a violation of academic integrity and may incur sanctions as outlined in the Illinois Student Code. Students should be aware the exam proctoring process involves screen recording of the computer. While AI will not be strictly prohibited for homework as they can be useful tools for studying and brainstorming, students are strongly encouraged to work on the homework themselves or collaborate with peers when appropriate. Students are required to submit their own work and answers. This improves learning outcomes and best prepares students for exams. Homework is designed to help students understand the content and develop skills. Students may be unprepared for exams if there is heavy reliance on AI.

Students are expected to follow the [University guidance on intellectual property](#). Students are not to share course content outside of the course.

Campus Holidays:

Instructors are expected to respond to student communications and grade homework assignments in a timely manner. Observed Campus Holidays, illness, personal obligations, or other circumstances outside the instructor's control may affect normal grading and response time. If you experience consistent and significant delays, please contact instruction@netmath.illinois.edu with your concerns.

Extensions:

Students in this course are eligible for up to two 1-month course extensions. There is a non-refundable \$350 fee for each approved extension. [Students should review the extension guidelines for more information.](#)

FERPA:

See [FERPA – Office of the Registrar](https://registrar.illinois.edu/academic-records/ferpa/) (<https://registrar.illinois.edu/academic-records/ferpa/>) for more information on the Family Educational Rights and Privacy Act (FERPA).

Any student who has suppressed their directory information pursuant to FERPA should self-identify to the instructor to ensure protection of the privacy of their attendance in this course.

Mental Health:

Significant stress, mood changes, excessive worry, substance/alcohol misuse or interferences in eating or sleep can have an impact on academic performance, social development, and emotional wellbeing. The University of Illinois offers a variety of confidential services including individual and group counseling, crisis intervention, psychiatric services, and specialized screenings which are covered through the Health Service Fee. If you or someone you know experiences any of the above mental health concerns, it is strongly encouraged to contact or visit any of the University's resources provided below. Getting help is a smart and courageous thing to do for yourself and for those who care about you.

[Counseling Center](#) or [\(217\) 333-3704](#)

[McKinley Health Center](#) or [\(217\) 333-2700](#)

[National Suicide Prevention Lifeline](#) or [\(800\) 273-8255](#)

[Rosecrance Crisis Line](#) or [\(217\) 359-4141](#) (available 24/7, 365 days a year)

If you are in immediate danger, call [911](#).

*This statement is approved by the [University of Illinois Counseling Center](#)

Sexual Misconduct Reporting Obligation:

The University of Illinois is committed to combating sexual misconduct. Faculty and staff members are required to report any instances of sexual misconduct to the University's Title IX Office. In turn, an individual with the [Title IX Office](#) will provide information about rights and options, including supportive measures, resources, the campus disciplinary process, and law enforcement options.

[Designated Confidential Employees](#): A list of the designated University employees who, as counselors, confidential advisors, and medical professionals, do not have this reporting responsibility and can maintain confidentiality.

Other information about [resources and incident reporting](#).